BUSINESS ARITHMETIC VERSUS ALGEBRA IN THE HIGH SCHOOL.

By George H. Van Tuyl.

In bringing this discussion before you to-day, it seems wise to state the reason for its selection. This explanation is offered, not in any sense as an apology for the subject or its discussion, but merely to state my position, lest I should be misunderstood.

When your President first wrote me he asked if I would speak on some phase of business arithmetic. I hardly knew what phase of business arithmetic would be interesting to a company of mathematics teachers. After some reflection it occurred to me that I had read an address delivered by Dr. David Eugene Smith, in which he stated as the first of a list of problems confronting mathematics teachers, this problem, which, to put it in his own words, reads: "The first of the great questions that confront us at the present time relates to the very existence of secondary mathematics in our curriculum." Dr. Smith admits he has no solution. I am not here claiming to solve problems which the gentleman quoted cannot solve. Nor is it in my thought to attack mathematics or mathematics teachers. We all recognize the value of mathematics in certain lines of work. The thoughts I may have to present are for the purpose of adding to the discussion. If from my remarks anything that will have a tendency to help solve the question of what are the most valuable subjects in a high school curriculum, can be gained, I shall be content.

In the first place let us keep clearly in mind the fact that our topic and its discussion have to do only with secondary or high school work. In the second place we must remember that only about five per cent. of our high school students ever go to college. Our discussion, then, must be carried on with a view to meeting the needs of those whose school days end at, or before, graduation from a secondary school. I claim we have no right in these days of the free public school to frame a course of study to meet the requirements of one student while we neglect the immediate and pressing need of nineteen others.

It is only proper, then, that at this point, we should formulate some notion of the purpose of the high school curriculum. For far too many years the main objective has been to prepare for college entrance. It is only within comparatively recent years that the demand to prepare for life's work has been heard and heeded. One by one subjects whose presence in the curriculum could not be justified even by their most ardent supporters, have been dropping out, and others of greater practical value have taken their places. The trend of events points clearly the way. The public is demanding that for the large investment in high school education there shall be a reasonably adequate return in the form of practical, usable knowledge in the possession of the boys and girls who are graduated from our high schools.

With these thoughts in mind, let me submit a question, trite though it may be, viz., "What is the educational value of secondary mathematics, or more particularly, of algebra, for students who do not go to college?" Prof. Young, in his "The Teaching of Mathematics," tells us that the study of mathematics exemplifies most typically, clearly and simply certain modes of thought," one of which modes, he says, "is the ability to grasp a situation, to seize the facts, and to perceive correctly the state of affairs." Of algebra, in particular, he mentions four functions only three of which will I call to your attention. (1) "To establish more carefully and to extend the theoretic processes of arithmetic. (2) To strengthen the pupil's power in computation by much practice as well as by the development of devices useful in computation. (3) To develop the equation and to apply it in the solution of problems of a wide range of interest, including large classes of problems often treated in arithmetic, as well as problems relative to geometry, to physics, and to other natural sciences."

It is unnecessary for me to attempt to tell you what are the purposes or the value of your own subject. The quotations just cited are for the purpose of establishing some basis of comparison between business arithmetic and algebra.

The first question that presents itself to my mind is, "Does mathematics in general, or algebra in particular, give one the 'ability to grasp a situation, seize the facts, and perceive correctly the state of affairs' better than does the study of business arith-

metic?" I am aware of the fact that an isolated case here and there proves but little, yet I would like to mention one or two conspicuous illustrations of the failure of mathematics to enable students to "grasp the situation." It was my privilege recently to give instruction in business arithmetic to two young ladies, one of whom had passed differential calculus, while the other was planning to specialize in mathematics for her Ph.D. degree. One of them brought me this problem: "A man bought two horses at the same price; he sold one of them at a loss of 10 per cent., but for the other one he received a sum sufficient to make up his loss on the first horse and 20 per cent. of that loss in addition. He thereby gained \$5.00. What was the cost of each horse? What was the loss on one, the gain on the other?" I do not recall the solution she presented with it. It was an attempt at an algebraic solution with the usual X for the unknown value. Aside from the original statement of "$X=$ the value of each horse," she had not a single correct equation in her solution, and needless to say, she did not arrive at a correct conclusion.

Let us apply a little business arithmetic to the problem in question. Note the conditions. Two horses cost the same amount; one was sold at a loss of 10 per cent., and the other at a sufficient gain to make up the loss on the first and 20 per cent. of that loss additional. The gain on both was \$5.00. The problem reduces to this simple equation: 20 per cent. of 10 per cent. of the cost of 1 horse = \$5.00. Solving we have 20 per cent. of 10 per cent. = 2 per cent.; that is 2 per cent. of the cost of one horse = \$5.00. Since 2 per cent. of the cost is \$5.00, the entire cost is 50 times \$5.00, or \$250. The remainder of the solution does not need discussion here. The problem as you perceive is a very simple one, and is solved mentally with ease. Surely mathematics through calculus did not give "ability to grasp a situation, to seize the facts, and to perceive correctly the state of affairs."

The other young lady was even less able to solve problems in business arithmetic, though she could talk glibly of quadratics, surds, negative exponents, irrational quantities, etc.

I have at the present time a class of young men who have had from two and one half to three years of high school mathematics, who were unable to solve or even to state, when they entered my

class, such a simple problem as this: "A house and lot cost \$5,000. If taxes, repairs and other expenses amount to \$180 per annum, what rent per month must the owner receive to clear 6 per cent. on his investment?" Had this been a condition afflicting only a small portion of the class, it might have been overlooked as the usual condition of affairs, but fully 75 per cent. of the class were unable to solve the problem quoted and others of like difficulty. This is a condition not at all peculiar to the present class, but it is one which confronts me from term to term. In the face of these conditions one begins to wonder what has become of the "ability to grasp a situation" which a study of mathematics is said to give.

Some years ago I taught business arithmetic in a school in which but very few of the students had ever studied any other branch of mathematics than arithmetic in the elementary school. These boys and girls did better work in arithmetic than do the boys I now have. Do not understand me to say that they did better work because they had no knowledge of mathematics; what I mean is that those who *have* had the mathematics do not show by their ability to solve problems that mathematics has helped them.

Recently I had the pleasure of visiting a school in a neighboring city. Naturally I visited an arithmetic class. The students were finishing their second year in high school. They had studied algebra the preceding year and a half. The arithmetic was being taught by the use of algebraic formulæ, logarithms, etc. I left the class with the firm conviction that if the year and a half spent in the study of algebra could have been devoted to business arithmetic these students would have been far better equipped to interpret and solve problems in the business world than they are with the knowledge of algebra they displayed.

What is the matter? Why is it that so uniformly the speaker has found students not possessing the ability which their study of mathematics is supposed to give them? Is it poor teaching, or is it possible that the study of mathematics does not give what is claimed for it? Let me venture an opinion. It looks this way to me: The student of algebra spends a year or more studying unknown quantities, negative quantities, surds, and to him a lot of things that are *ab*surd, and at the end of that time he has

mixed up in his mind a lot of unknown " unknowns." How many high school students can give an intelligent exposition of imaginary quantities or complex roots? These and many other topics common in algebra are entirely outside the experience of the pupil. There is little or nothing in his mind to which he can relate or attach the new matter. He has no means of classifying the new information he is receiving. The result is a jumble, and consequently, when the student wishes to apply his knowledge to the solution of practical problems he is at a loss to know how to proceed. In other words, the time required to train a student to solve problems intelligently by the algebraic process is greater than is allowed for such training in the high school, or is warranted by the benefits derived therefrom.

A second reason, in my opinion, is that in the study of algebra, too much attention is given to theorizing on possible solutions for impractical conditions, and to solving equations that are already prepared for the student, or, to put it differently, too *little* attention is given to solving practical problems. In one of the recently published algebras containing 447 pages there are less than 500 problems for solution. By problems I mean exercises requiring the student to formulate his own equations from the data given. The great majority of the problems are of no value save as exercises in calculation. They are made for no other purpose than to fit the theories that have been discussed in the earlier parts of the chapters. A student may be skillful in solving problems in simultaneous equations or in quadratics, but such skill does not aid him materially in solving business problems.

Professor Young says of algebra that its function is "to establish more carefully and to extend the theoretic processes of arithmetic." I have yet to find any process in arithmetic required in business that cannot be sufficiently and thoroughly established without resorting to algebraic processes. The demand of the day is not for theoretic processes, but for practical processes. The process that produces results the most quickly and the most easily is the one that wins. A study of negative quantities is not necessary to convince a boy that, if he earns $10.00 and spends $5.00 of it, he has $5.00 left; or if he starts from his home and walks 8 miles north and then 4 miles south, he is 4 miles from his

home. Neither does he need to study simultaneous equations to solve this problem: "A laborer engaged to work 48 days at $2.00 a day and his board. But for every day he was idle he was to pay $1.00 for his board. At the end of the time he received only $42.00. How many days did he work?" All such a problem needs is the application of a little common sense, and it can be solved mentally far more quickly and easily than by simultaneous equations.

The third function of algebra, according to Professor Young is "to develop the equation and to apply it in the solution of problems, etc." No one favors the use of the equation in solving problems more than does the speaker. I fear too much attention is given to developing the equation and too little to its application. A business arithmetic was recently published, in which the author, a former teacher of mathematics, tried to feature the algebraic equation as a mode of calculation. Four pages were used in developing the equation, and then he left it suspended in mid-air and did not apply it to a single problem.

Enough has been said concerning the algebraic side of our topic. Let us now give some thought to business arithmetic. The question is sometimes asked, "What is business arithmetic? How does it differ from arithmetic? I would answer the first question by saying, business arithmetic is the arithmetic of business. So far as the calculation side of the subject is concerned it has for its object the solving of problems and the making of calculations by up-to-date business methods, which is only another way of saying that solutions and calculations are made by the easiest, simplest, and shortest methods possible. Ordinary arithmetic is coming more and more to approach this standard. The difference between the two has been more in the kind of problems and in the method of calculation, than in anything else.

Business arithmetic has, then, as one of its objectives, skill in calculation. Too many people seem to think that skill in calculation is the main objective, that the business arithmetic teacher is training only for mechanical celerity, and that the business arithmetic course does not develop power "to grasp a situation, to seize the facts, and to perceive correctly the state of affairs." Far be it from me to hold up any such standard. In the first place it would be ridiculous, and in the next place its attainment

would be impossible. Skill in calculation (I shall use the term calculation not merely for the mechanical process of performing the fundamental operations, but for the entire process of interpreting and solving problems) is not a cause of power, but the result of power. Skill in calculation depends upon ability to interpret a problem, power or ability to apply the principles of business and of arithmetic to the problem, and ability to see the relation of numbers. Facility in calculation is the outward evidence of inward power. A second objective, then, of teaching business arithmetic, is the development of power. How shall we develop the power that leads to skillful calculation? You know as well as I do that pupils entering high school are not able to calculate with ease and accuracy. I have tried to tell you that when you turn them over to me for the study of business arithmetic they cannot interpret problems. Can we make ready calculators of our boys and girls by the study of business arithmetic? I believe we can. What are the requisites? First, time. Sufficient time must be given in which to teach the subject. The idea that some people have that in eight years of elementary school a boy has learned or ought to have learned all the arithmetic he needs, is indicative to my mind of only one thing, and that is that the person holding such ideas is entirely ignorant of the content of arithmetic. In many high schools one half a year is given to the subject. That time is entirely inadequate. Daily recitations for from one to two years are needed in the first half of the course. Later daily recitations for at least one half a year are required to take up topics too advanced for earlier discussion. A second requisite is qualified teachers. Here again, an erroneous impression seems to prevail. Owing to an incorrect notion concerning business arithmetic, many seem to think that anybody can teach the subject. In some schools business arithmetic is taught by teachers of modern language, or of English, or of any other department having a teacher to spare, while in other schools, the subject has been turned over to the mathematics department. With all due respect to the teachers of mathematics, I believe such a method of procedure to be unwise, and that for one reason only—mathematics teachers are not teachers of business arithmetic. Is it any wonder we get poor results?

I have not the authority to grant enough time for the study of business arithmetic, nor can I provide a sufficient number of qualified teachers to teach the subject. So let us consider a third requisite for the development of power. It is method. It is not in my thought to discuss methods as laid down in the pedagogical books, but to outline if I can by specific instances how we may proceed to develop power. I have yet to find the class that did not need drill in the fundamentals. Classes as a whole are slow and inaccurate; they are slow because they are using "long hand" methods of calculating when they ought to be using "short hand" methods. They are inaccurate partly because they are using cumbersome methods, and partly because they do not realize the importance of accuracy.

The first thing to do is to arouse interest in and enthusiasm for the subject. One way, and I believe the best way, of doing this is to start the class on such a topic as aliquot parts. Use simple exercises at first, explaining reasons and underlying principles. As the exercises increase in difficulty it will be observed that the subject divides into two parts, viz., direct aliquots or aliquants, as $.33\frac{1}{3}$ and $.37\frac{1}{2}$, and the indirect aliquots and aliquants, as $.11\frac{1}{4}$, $.13\frac{1}{3}$ and $.13\frac{3}{4}$. It is just as easy to multiply by $.13\frac{1}{3}$ as by $.12\frac{1}{2}$. As soon as a pupil begins to see the opportunities there are for abbreviated forms of calculation, and begins to study the relation of numbers for himself, he is on the high road to power that gives skill in calculation. There are a large number of direct and indirect aliquots and aliquants, but to use them intelligently requires a careful study of the relation of numbers. Let me give just a few instances to illustrate my meaning. Probably the most common numbers of which the aliquot parts are used are 100 and 60—100 as 100 cents in a dollar, or as 100 per cent. in percentage, and 60 in interest calculations. Let us consider some of the parts of 100 cents or $1.00. Let it be required to find the value of 48 articles at $11\frac{1}{4}$c. each. $11\frac{1}{4}$c. is equal to 10c. plus $\frac{1}{8}$ of 10c. Taking advantage of the commutative law of multiplication all we need to do is to point off 1 place in the 48 and add $\frac{1}{8}$ of the result, which gives a value of $5.40. If the problem were to find the value of 48 articles at $13\frac{3}{4}$c. each, we would first think of $13\frac{3}{4}$c. as $12\frac{1}{2}$c. plus $\frac{1}{10}$ of $12\frac{1}{2}$c. The result, then, is found by taking

$\frac{1}{8}$ of 48, which is 6, and adding $\frac{1}{10}$ of the 6 to itself, which gives 6.60, that is $6.60. Again let it be required to find the value of 48 articles at $13\frac{1}{2}$c. $13\frac{1}{2}$c. equals $12\frac{1}{2}$c. plus 1c. The result is equal to $\frac{1}{8}$ of 48, which is 6 (that is $6.00) plus 48c., hence $6.48. You will observe that the prime object in such exercises as these is not the mere finding of the value of a given number of articles at a given price, but a study in the relation of numbers, which as I have already stated is one of the requisites of skill in calculation.

Similarly, there is great opportunity for a study of the relation of numbers in the use of the aliquots and multiples of 60. Let me give an illustration or two. For the purposes of interest calculations 60 days are regarded as equal to two months. Two months is $\frac{1}{6}$ of a year. Hence at 6 per cent. interest, the interest on $1.00 for 2 months (or 60 days) is $.01. Since the interest on $1.00 for 60 days at 6 per cent. is $.01, it is $.01 on each of any number of dollars for the same time and rate. Therefore to find the interest on any sum of money for 60 days at 6 per cent. point off 2 places, or remove the decimal point 2 places to the left. Having the interest for 60 days, it can be readily found for any number of days. For 30 days it would be half of the interest for 60 days; for 15 days, $\frac{1}{4}$; for 20 days, $\frac{1}{3}$; for 75 days, add $\frac{1}{4}$ of the interest at 60 days to itself; for 16 days, take $\frac{1}{6}$ for 10 days, and then $\frac{1}{10}$ for 6 days, and add the results for 16 days, etc. By the commutative law of multiplication the interest on $600 for 79 days at 6 per cent. is found by pointing off 1 place in the days, hence $7.90. If time permitted I could give almost endless illustrations of the use of this principle.

Now let us consider a topic in fractions as a study in the relation of numbers. Suppose it is desired to multiply $16\frac{1}{4}$ by $8\frac{1}{2}$. Probably 8 students out of 10 who enter high school will, if given this operation to perform, reduce each mixed number to an improper fraction. They will then multiply 65 by 17, and divide the product by 8, using in most cases the long division process. What a waste of time, of effort, of opportunity! How much better is the four step process! Look at the numbers. Think the product of the fractions, of the cross products, and finally the products of the integers, adding the parts as you proceed. The result is $138\frac{1}{8}$. Once more, let us multiply $7\frac{1}{3}$ by $5\frac{1}{3}$. Think

the product of the fractions. The cross products ($\frac{1}{3}$ of 5) + ($\frac{1}{3}$ of 7) = $\frac{1}{3}$ of 12; the product of the integers is 35, hence $39\frac{1}{9}$, as the completed product. This is the kind of drill that will develop power that will be usable in the solution of problems. But students must be directed in this work; they will not discover these methods for themselves.

The topics illustrated, together with many others that might be mentioned, have to do with the fundamental operations. Proper kind of drill in these subjects arouses interest in and creates a liking for arithmetic. Calculations of this kind require thought, and keep the pupil ever on the alert for new, short methods. Every operation becomes a problem in method of calculation. Soon the pupil becomes an independent investigator in the realm of practical methods of calculation. This is the kind of work from which mental discipline comes.

Let us now come to the question of problems. We observed that, in algebra, problems were made to fit certain theories of calculation regardless of the content of the problem. It is "putting the cart before the horse." In business arithmetic, the chief emphasis is laid on providing practical problems, and then devising practical methods of solving them. To solve problems successfully requires a knowledge of, or the ability to do, four things. First, one must be able to read plain, simple English understandingly. Many a failure in arithmetic, and in mathematics, has its roots right here. If one cannot from the printed page ascertain what data are given, and what result is required, his case is hopeless until such defect is remedied. Second, one must be able to perform the fundamental operations with ease and accuracy. We have already spoken of this phase of the work, and it is assumed that, by the time problems are presented for solution, students will be somewhat proficient in the mechanical processes required. The third and fourth things necessary for successful solution of problems, I shall discuss together, as I consider them indissolubly connected. They are the fundamental principles of business and the fundamental principles of arithmetic. As the principles of business let me mention two or three merely as illustrations: Cost is the basis on which gain or loss is reckoned; commission is computed on the prime cost or the gross sales; bank discount is reckoned on the maturity

value of a note for the exact number of days (with very few exceptions) from date of discount to date of maturity. A knowledge of these principles and many others that might be cited is absolutely necessary to interpret problems. As an illustration of the principles of arithmetic, I will mention but one. The product of two factors divided by either of them will give the other one. This one principle underlies every business problem involving a division. The application of this principle results in the use of the equation, the development and application of which Prof. Young says is one of the functions of algebra. Personally I see no need for algebra in this connection.

Let me illustrate the use of the principles and of the equation in a few simple problems. Of a consignment of eggs 5 per cent. were broken. For the remainder \$39.90 was received at 35c. per dozen. How many dozen eggs were in the consignment? If 5 per cent. of the eggs were broken, 95 per cent. of them remained to be sold. The problem reduces to two equations:

1. ? × \$.35 = 39.90.

Dividing the product, \$39.90 by the factor, \$.35, gives 114, the number of dozen sold.

2. 95 per cent. of ? doz. = 114 doz.

Dividing 114 doz. by .95 gives 120 doz., the desired result.

Take a second problem: A jobber buys hats at \$48 less 20 per cent. and 16⅔ per cent. per dozen from the manufacturer. At what price per dozen should he mark them to gain 35 per cent. after allowing 16⅔ per cent. and 10 per cent. to his customers?

1. The series 20 per cent. and 16⅔ per cent. = a single discount of 33⅓ per cent.

2. 66⅔ per cent. of \$48 = \$32, net cost per dozen.

3. 35 per cent. of \$32 = \$11.20, gain.

4. \$32 + \$11.20 = \$43.20, net selling price.

5. The series 16⅔ per cent. and 10 per cent. = a single discount of 25 per cent.

Hence the net selling price equals 75 per cent. of the marked price.

6. 75 per cent. of M. P. = \$43.20.

7. \$43.20 ÷ .75 = \$57.60, the marked price.

All through this problem are interwoven the principles of business and the principles of arithmetic. The solution is given in full to illustrate the use of the equation. Personally, I prefer a much briefer solution. It is this:

3) $48
16, discount.
32, net cost.
11.20, gain (35 per cent. of net cost).
3) 43.20, net selling price.
14.40, discount to purchasers.
$57.60, marked price.

Every principle and every equation used in the other solution is here used but in a shorter, more practical manner. When the student can solve problems by this shortened process, it is an evidence of clear thinking and careful application—an evidence of inward power.

A man desires to buy 150 shares of stock paying 6 per cent. dividends at a price to yield 5 per cent. on his investment. At what price should he buy the stock, and how much will the 150 shares cost him? (Make no allowance for brokerage.)

1. 6 per cent. of $100 = $6, dividend on 1 share.
2. 5 per cent. of cost of 1 share = $6.
3. $6 ÷ .05 = $120, cost of 1 share.
4. 150 × $120 = $18,000, cost of 150 shares.

Here again we find the use of both the principles of business and of arithmetic.

If our students possess the ability to read intelligently, can add, subtract, multiply, and divide correctly, and have a knowledge of business principles and customs, and of arithmetical principles, they can solve any business problem presented to them. In two of these particulars, in my opinion, algebra fails. It gives no training in business principles, nor in the proper arithmetical principles. As to the equation, it can just as well be taught and used in arithmetic as in algebra.

In conclusion, then, let me say that, so far as my experience goes, I do not find that a study of mathematics, or of algebra in particular, gives ability to solve business problems. This is due chiefly, I believe, to the following: First, in algebra, symbols

are used for the thing or number for which the symbol stands. I see no advantage for business purposes in using an imaginary thing or number for the real, concrete thing itself, as represented by its correct name or value. In the second place the problems in algebra are artificial, impractical, and largely useless for any business purpose. And thirdly, one does not learn in algebra the fundamental principles of business or of arithmetic as applied to business, nor does he learn the abbreviated methods of calculation so important in the business world.

What shall we say then; shall algebra continue to be required in the high school that mathematics may abound, or shall we lay aside our prejudices, and investigate the merits of business arithmetic? Let me be a radical for just a moment, and suggest that business arithmetic be made a required subject throughout the first year of the high school. If necessary, eliminate part of the mathematics, and begin the study of algebra in the second year of the course. This would give those students who leave at or before the end of the first year something practical that they could use upon leaving school. It would also give a far better type of student, on the average, for the study of algebra. This fact, together with the fact that they would have a better understanding of arithmetic, which is the basis of algebra, would largely compensate for the loss of the first year in the study of mathematics. I do not expect you to agree with me in this, but I firmly believe it would result in decided advantage to our students.

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